NAG Toolbox for MATLAB

g01fe

1 Purpose

g01fe returns the deviate associated with the given lower tail probability of the beta distribution, via the function name.

2 Syntax

```
[result, ifail] = g01fe(p, a, b, tol)
```

3 Description

The deviate, β_p, associated with the lower tail probability, p, of the beta distribution with parameters a and b is defined as the solution to

$$P\left(B \leq \beta_p : a, b\right) = p = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^{\beta_p} B^{a-1}(1-B)^{b-1}\, dB, \qquad 0 \leq \beta_p \leq 1; a, b > 0.$$

The algorithm is a modified version of the Newton–Raphson method, following closely that of Cran *et al.* 1977.

An initial approximation, β_0, to β_p is found (see Cran *et al.* 1977), and the Newton–Raphson iteration

$$\beta_i = \beta_{i-1} - \frac{f(\beta_{i-1})}{f'(\beta_{i-1})},$$

where $f(\beta) = \mathbf{p}(\mathbf{b} \leq \beta : a, b) - p$ is used, with modifications to ensure that β remains in the range $(0, 1)$.

4 References

Cran G W, Martin K J and Thomas G E 1977 Algorithm AS109. Inverse of the incomplete beta function ratio *Appl. Statist.* **26** 111–114

Hastings N A J and Peacock J B 1975 *Statistical Distributions* Butterworths

5 Parameters

5.1 Compulsory Input Parameters

1: **p – double scalar**

p, the probability from the required beta distribution.

Constraint: $0.0 \leq \mathbf{p} \leq 1.0$.

2: **a – double scalar**

a, the first parameter of the required beta distribution.

Constraint: $0.0 < \mathbf{a} \leq 10^6$.

3: **b – double scalar**

b, the second parameter of the required beta distribution.

Constraint: $0.0 < \mathbf{b} \leq 10^6$.

4: **tol – double scalar**

The relative accuracy required by you in the result. If g01fe is entered with **tol** greater than or equal to 1.0 or less than 10 times ***machine precision*** (see x02aj), then the value of $10 \times$ ***machine precision*** is used instead.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01fe may return useful information for one or more of the following detected errors or warnings.

If on exit **ifail** $= 1$ or 2, then g01fe returns 0.0.

ifail $= 1$

On entry, $\mathbf{p} < 0.0$,
or $\mathbf{p} > 1.0$.

ifail $= 2$

On entry, $\mathbf{a} \leq 0.0$,
or $\mathbf{a} > 10^6$,
or $\mathbf{b} \leq 0.0$,
or $\mathbf{b} > 10^6$.

ifail $= 3$

There is doubt concerning the accuracy of the computed result. 100 iterations of the Newton–Raphson method have been performed without satisfying the accuracy criterion (see Section 7). The result should be a reasonable approximation of the solution.

ifail $= 4$

Requested accuracy not achieved when calculating beta probability. The result should be a reasonable approximation to the correct solution. You should try setting **tol** larger.

7 Accuracy

The required precision, given by **tol**, should be achieved in most circumstances.

8 Further Comments

The typical timing will be several times that of g01ee and will be very dependent on the input parameter values. See g01ee for further comments on timings.

9 Example

```
p = 0.5;
a = 1;
b = 2;
tol = 0;
[result, ifail] = g01fe(p, a, b, tol)
```

```
result =
    0.2929
ifail =
           0
```